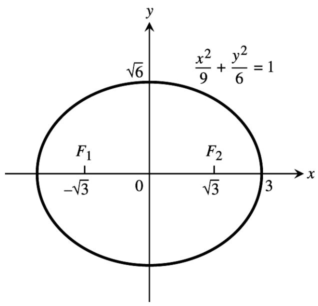
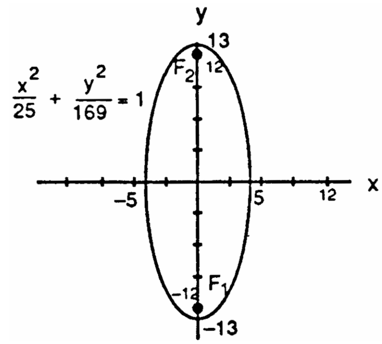
23. $6x^2 + 9y^2 = 54 \Rightarrow \frac{x^2}{9} + \frac{y^2}{6} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{9 - 6} = \sqrt{3}$



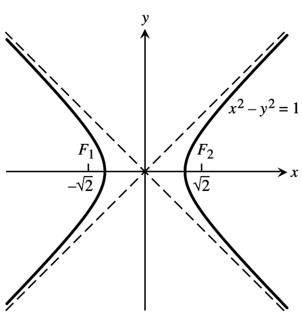
24. $169x^2 + 25y^2 = 4225 \Rightarrow \frac{x^2}{25} + \frac{y^2}{169} = 1$
$\Rightarrow c = \sqrt{a^2 - b^2} = \sqrt{169 - 25} = 12$



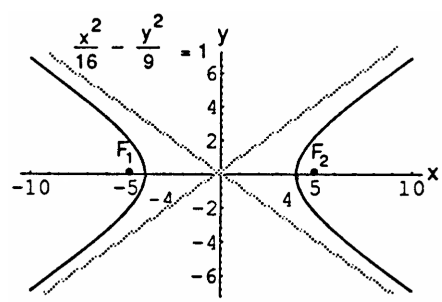
25. Foci: $\left(\pm\sqrt{2}, 0\right)$, Vertices: $(\pm 2, 0) \Rightarrow a = 2, c = \sqrt{2} \Rightarrow b^2 = a^2 - c^2 = 4 - \left(\sqrt{2}\right)^2 = 2 \Rightarrow \frac{x^2}{4} + \frac{y^2}{2} = 1$

26. Foci: $(0, \pm 4)$, Vertices: $(0, \pm 5) \Rightarrow a = 5, c = 4 \Rightarrow b^2 = 25 - 16 = 9 \Rightarrow \frac{x^2}{9} + \frac{y^2}{25} = 1$

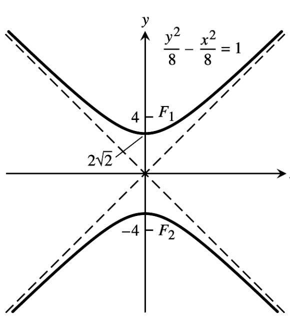
27. $x^2 - y^2 = 1 \Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{1 + 1} = \sqrt{2}$; asymptotes are $y = \pm x$



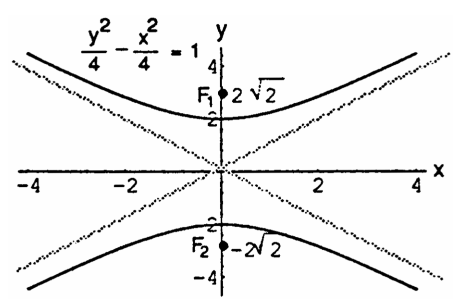
28. $9x^2 - 16y^2 = 144 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$
$\Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{16 + 9} = 5$;
asymptotes are $y = \pm \frac{3}{4}x$



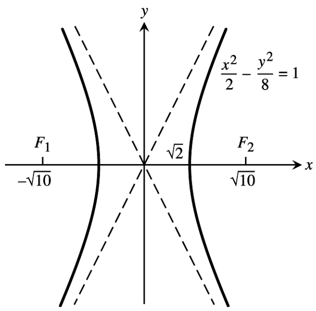
29. $y^2 - x^2 = 8 \Rightarrow \frac{y^2}{8} - \frac{x^2}{8} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{8 + 8} = 4$; asymptotes are $y = \pm x$



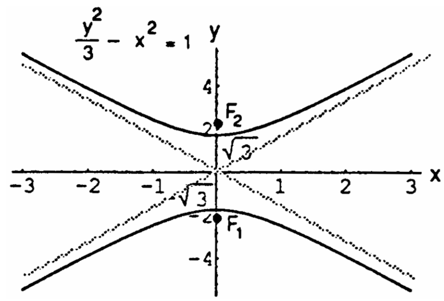
30. $y^2 - x^2 = 4 \Rightarrow \frac{y^2}{4} - \frac{x^2}{4} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{4 + 4} = 2\sqrt{2}$; asymptotes are $y = \pm x$



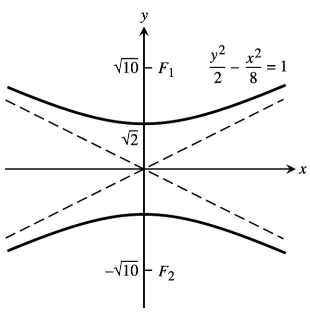
31. $8x^2 - 2y^2 = 16 \Rightarrow \frac{x^2}{2} - \frac{y^2}{8} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{2+8} = \sqrt{10}$; asymptotes are $y = \pm 2x$



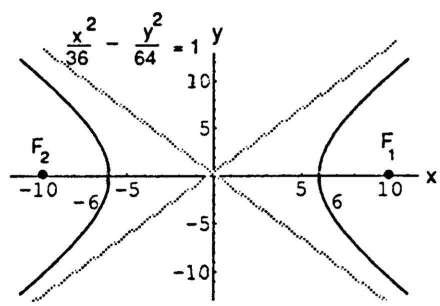
32. $y^2 - 3x^2 = 3 \Rightarrow \frac{y^2}{3} - x^2 = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{3+1} = 2$; asymptotes are $y = \pm \sqrt{3}x$



33. $8y^2 - 2x^2 = 16 \Rightarrow \frac{y^2}{2} - \frac{x^2}{8} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{2+8} = \sqrt{10}$; asymptotes are $y = \pm \frac{x}{2}$



34. $64x^2 - 36y^2 = 2304 \Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{36+64} = 10$; asymptotes are $y = \pm \frac{4}{3}$



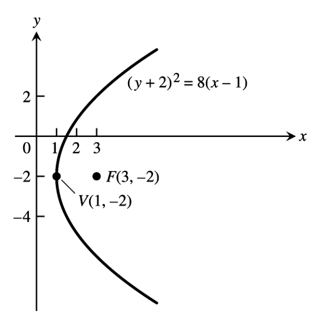
35. Foci: $\left(0, \pm\sqrt{2}\right)$, Asymptotes: $y = \pm x \Rightarrow c = \sqrt{2}$ and $\frac{a}{b} = 1 \Rightarrow a = b \Rightarrow c^2 = a^2 + b^2 = 2a^2 \Rightarrow 2 = 2a^2$
$\Rightarrow a = 1 \Rightarrow b = 1 \Rightarrow y^2 - x^2 = 1$

36. Foci: $(\pm 2, 0)$, Asymptotes: $y = \pm \frac{1}{\sqrt{3}} x \Rightarrow c = 2$ and $\frac{b}{a} = \frac{1}{\sqrt{3}} \Rightarrow b = \frac{a}{\sqrt{3}} \Rightarrow c^2 = a^2 + b^2 = a^2 + \frac{a^2}{3} = \frac{4a^2}{3}$
$\Rightarrow 4 = \frac{4a^2}{3} \Rightarrow a^2 = 3 \Rightarrow a = \sqrt{3} \Rightarrow b = 1 \Rightarrow \frac{x^2}{3} - y^2 = 1$

37. Vertices: $(\pm 3, 0)$, Asymptotes: $y = \pm \frac{4}{3} x \Rightarrow a = 3$ and $\frac{b}{a} = \frac{4}{3} \Rightarrow b = \frac{4}{3}(3) = 4 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

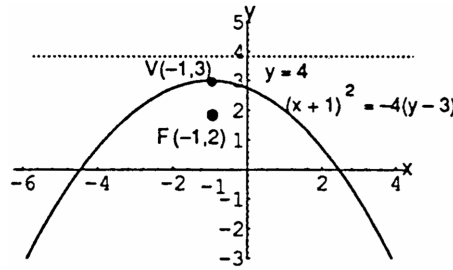
38. Vertices: $(0, \pm 2)$, Asymptotes: $y = \pm \frac{1}{2} x \Rightarrow a = 2$ and $\frac{a}{b} = \frac{1}{2} \Rightarrow b = 2(2) = 4 \Rightarrow \frac{y^2}{4} - \frac{x^2}{16} = 1$

39. (a) $y^2 = 8x \Rightarrow 4p = 8 \Rightarrow p = 2 \Rightarrow$ directrix is $x = -2$, focus is $(2, 0)$, and vertex is $(0, 0)$; therefore the new directrix is $x = -1$, the new focus is $(3, -2)$, and the new vertex is $(1, -2)$



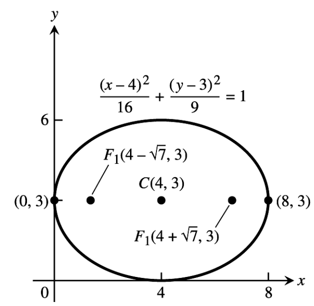
40. (a) $x^2 = -4y \Rightarrow 4p = 4 \Rightarrow p = 1 \Rightarrow$ directrix is $y = 1$, focus is $(0, -1)$, and vertex is $(0, 0)$; therefore the new directrix is $y = 4$, the new focus is $(-1, 2)$, and the new vertex is $(-1, 3)$

(b)



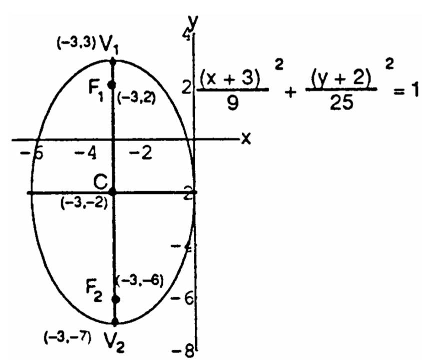
41. (a) $\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(-4, 0)$ and $(4, 0)$; $c = \sqrt{a^2 - b^2} = \sqrt{7} \Rightarrow$ foci are $\left(\sqrt{7}, 0\right)$ and $\left(-\sqrt{7}, 0\right)$; therefore the new center is $(4, 3)$, the new vertices are $(0, 3)$ and $(8, 3)$, and the new foci are $\left(4 \pm \sqrt{7}, 3\right)$

(b)



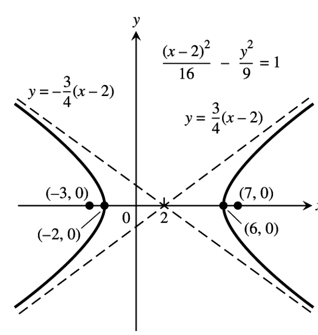
42. (a) $\frac{x^2}{9} + \frac{y^2}{25} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(0, 5)$ and $(0, -5)$; $c = \sqrt{a^2 - b^2} = \sqrt{16} = 4 \Rightarrow$ foci are $(0, 4)$ and $(0, -4)$; therefore the new center is $(-3, -2)$, the new vertices are $(-3, 3)$ and $(-3, -7)$, and the new foci are $(-3, 2)$ and $(-3, -6)$

(b)



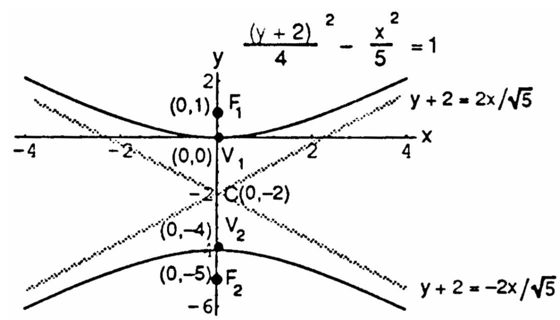
43. (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(-4, 0)$ and $(4, 0)$, and the asymptotes are $\frac{x}{4} = \pm \frac{y}{3}$ or $y = \pm \frac{3x}{4}$; $c = \sqrt{a^2 + b^2} = \sqrt{25} = 5 \Rightarrow$ foci are $(-5, 0)$ and $(5, 0)$; therefore the new center is $(2, 0)$, the new vertices are $(-2, 0)$ and $(6, 0)$, the new foci are $(-3, 0)$ and $(7, 0)$, and the new asymptotes are $y = \pm \frac{3(x-2)}{4}$

(b)



44. (a) $\frac{y^2}{4} - \frac{x^2}{5} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(0, -2)$ and $(0, 2)$, and the asymptotes are $\frac{y}{2} = \pm \frac{x}{\sqrt{5}}$ or $y = \pm \frac{2x}{\sqrt{5}}$; $c = \sqrt{a^2 + b^2} = \sqrt{9} = 3 \Rightarrow$ foci are $(0, 3)$ and $(0, -3)$; therefore the new center is $(0, -2)$, the new vertices are $(0, -4)$ and $(0, 0)$, the new foci are $(0, 1)$ and $(0, -5)$, and the new asymptotes are $y + 2 = \pm \frac{2x}{\sqrt{5}}$

(b)



45. $y^2 = 4x \Rightarrow 4p = 4 \Rightarrow p = 1 \Rightarrow$ focus is $(1, 0)$, directrix is $x = -1$, and vertex is $(0, 0)$; therefore the new vertex is $(-2, -3)$, the new focus is $(-1, -3)$, and the new directrix is $x = -3$; the new equation is $(y + 3)^2 = 4(x + 2)$

46. $y^2 = -12x \Rightarrow 4p = 12 \Rightarrow p = 3 \Rightarrow$ focus is $(-3, 0)$, directrix is $x = 3$, and vertex is $(0, 0)$; therefore the new vertex is $(4, 3)$, the new focus is $(1, 3)$, and the new directrix is $x = 7$; the new equation is $(y - 3)^2 = -12(x - 4)$

47. $x^2 = 8y \Rightarrow 4p = 8 \Rightarrow p = 2 \Rightarrow$ focus is $(0, 2)$, directrix is $y = -2$, and vertex is $(0, 0)$; therefore the new vertex is $(1, -7)$, the new focus is $(1, -5)$, and the new directrix is $y = -9$; the new equation is $(x - 1)^2 = 8(y + 7)$

48. $x^2 = 6y \Rightarrow 4p = 6 \Rightarrow p = \frac{3}{2} \Rightarrow$ focus is $\left(0, \frac{3}{2}\right)$, directrix is $y = -\frac{3}{2}$, and vertex is $(0, 0)$; therefore the new vertex is $(-3, -2)$, the new focus is $\left(-3, -\frac{1}{2}\right)$, and the new directrix is $y = -\frac{7}{2}$; the new equation is $(x + 3)^2 = 6(y + 2)$

49. $\frac{x^2}{6} + \frac{y^2}{9} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(0, 3)$ and $(0, -3)$; $c = \sqrt{a^2 - b^2} = \sqrt{9 - 6} = \sqrt{3} \Rightarrow$ foci are $\left(0, \sqrt{3}\right)$ and $\left(0, -\sqrt{3}\right)$; therefore the new center is $(-2, -1)$, the new vertices are $(-2, 2)$ and $(-2, -4)$, and the new foci are $\left(-2, -1 \pm \sqrt{3}\right)$; the new equation is $\frac{(x+2)^2}{6} + \frac{(y+1)^2}{9} = 1$

50. $\frac{x^2}{2} + y^2 = 1 \Rightarrow$ center is $(0, 0)$, vertices are $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$; $c = \sqrt{a^2 - b^2} = \sqrt{2 - 1} = 1 \Rightarrow$ foci are $(-1, 0)$ and $(1, 0)$; therefore the new center is $(3, 4)$, the new vertices are $\left(3 \pm \sqrt{2}, 4\right)$, and the new foci are $(2, 4)$ and $(4, 4)$; the new equation is $\frac{(x-3)^2}{2} + (y - 4)^2 = 1$

51. $\frac{x^2}{3} + \frac{y^2}{2} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $\left(\sqrt{3}, 0\right)$ and $\left(-\sqrt{3}, 0\right)$; $c = \sqrt{a^2 - b^2} = \sqrt{3 - 2} = 1 \Rightarrow$ foci are $(-1, 0)$ and $(1, 0)$; therefore the new center is $(2, 3)$, the new vertices are $\left(2 \pm \sqrt{3}, 3\right)$, and the new foci are $(1, 3)$ and $(3, 3)$; the new equation is $\frac{(x-2)^2}{3} + \frac{(y-3)^2}{2} = 1$

52. $\frac{x^2}{16} + \frac{y^2}{25} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(0, 5)$ and $(0, -5)$; $c = \sqrt{a^2 - b^2} = \sqrt{25 - 16} = 3 \Rightarrow$ foci are $(0, 3)$ and $(0, -3)$; therefore the new center is $(-4, -5)$, the new vertices are $(-4, 0)$ and $(-4, -10)$, and the new foci are $(-4, -2)$ and $(-4, -8)$; the new equation is $\frac{(x+4)^2}{16} + \frac{(y+5)^2}{25} = 1$

53. $\frac{x^2}{4} - \frac{y^2}{5} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(2, 0)$ and $(-2, 0)$; $c = \sqrt{a^2 + b^2} = \sqrt{4 + 5} = 3 \Rightarrow$ foci are $(3, 0)$ and $(-3, 0)$; the asymptotes are $\pm \frac{x}{2} = \frac{y}{\sqrt{5}} \Rightarrow y = \pm \frac{\sqrt{5}x}{2}$; therefore the new center is $(2, 2)$, the new vertices are

$(4, 2)$ and $(0, 2)$, and the new foci are $(5, 2)$ and $(-1, 2)$; the new asymptotes are $y - 2 = \pm\frac{\sqrt{5}(x-2)}{2}$; the new equation is $\frac{(x-2)^2}{4} - \frac{(y-2)^2}{5} = 1$

54. $\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(4, 0)$ and $(-4, 0)$; $c = \sqrt{a^2 + b^2} = \sqrt{16 + 9} = 5 \Rightarrow$ foci are $(-5, 0)$ and $(5, 0)$; the asymptotes are $\pm\frac{x}{4} = \frac{y}{3} \Rightarrow y = \pm\frac{3x}{4}$; therefore the new center is $(-5, -1)$, the new vertices are $(-1, -1)$ and $(-9, -1)$, and the new foci are $(-10, -1)$ and $(0, -1)$; the new asymptotes are $y + 1 = \pm\frac{3(x+5)}{4}$; the new equation is $\frac{(x+5)^2}{16} - \frac{(y+1)^2}{9} = 1$

55. $y^2 - x^2 = 1 \Rightarrow$ center is $(0, 0)$, vertices are $(0, 1)$ and $(0, -1)$; $c = \sqrt{a^2 + b^2} = \sqrt{1 + 1} = \sqrt{2} \Rightarrow$ foci are $\left(0, \pm\sqrt{2}\right)$; the asymptotes are $y = \pm x$; therefore the new center is $(-1, -1)$, the new vertices are $(-1, 0)$ and $(-1, -2)$, and the new foci are $\left(-1, -1 \pm \sqrt{2}\right)$; the new asymptotes are $y + 1 = \pm(x + 1)$; the new equation is $(y + 1)^2 - (x + 1)^2 = 1$

56. $\frac{y^2}{3} - x^2 = 1 \Rightarrow$ center is $(0, 0)$, vertices are $\left(0, \sqrt{3}\right)$ and $\left(0, -\sqrt{3}\right)$; $c = \sqrt{a^2 + b^2} = \sqrt{3 + 1} = 2 \Rightarrow$ foci are $(0, 2)$ and $(0, -2)$; the asymptotes are $\pm x = \frac{y}{\sqrt{3}} \Rightarrow y = \pm\sqrt{3}x$; therefore the new center is $(1, 3)$, the new vertices are $\left(1, 3 \pm \sqrt{3}\right)$, and the new foci are $(1, 5)$ and $(1, 1)$; the new asymptotes are $y - 3 = \pm\sqrt{3}(x - 1)$; the new equation is $\frac{(y-3)^2}{3} - (x - 1)^2 = 1$

57. $x^2 + 4x + y^2 = 12 \Rightarrow x^2 + 4x + 4 + y^2 = 12 + 4 \Rightarrow (x + 2)^2 + y^2 = 16$; this is a circle: center at $C(-2, 0)$, $a = 4$

58. $2x^2 + 2y^2 - 28x + 12y + 114 = 0 \Rightarrow x^2 - 14x + 49 + y^2 + 6y + 9 = -57 + 49 + 9 \Rightarrow (x - 7)^2 + (y + 3)^2 = 1$; this is a circle: center at $C(7, -3)$, $a = 1$

59. $x^2 + 2x + 4y - 3 = 0 \Rightarrow x^2 + 2x + 1 = -4y + 3 + 1 \Rightarrow (x + 1)^2 = -4(y - 1)$; this is a parabola: $V(-1, 1)$, $F(-1, 0)$

60. $y^2 - 4y - 8x - 12 = 0 \Rightarrow y^2 - 4y + 4 = 8x + 12 + 4 \Rightarrow (y - 2)^2 = 8(x + 2)$; this is a parabola: $V(-2, 2)$, $F(0, 2)$

61. $x^2 + 5y^2 + 4x = 1 \Rightarrow x^2 + 4x + 4 + 5y^2 = 5 \Rightarrow (x + 2)^2 + 5y^2 = 5 \Rightarrow \frac{(x+2)^2}{5} + y^2 = 1$; this is an ellipse: the center is $(-2, 0)$, the vertices are $\left(-2 \pm \sqrt{5}, 0\right)$; $c = \sqrt{a^2 - b^2} = \sqrt{5 - 1} = 2 \Rightarrow$ the foci are $(-4, 0)$ and $(0, 0)$

62. $9x^2 + 6y^2 + 36y = 0 \Rightarrow 9x^2 + 6\left(y^2 + 6y + 9\right) = 54 \Rightarrow 9x^2 + 6(y + 3)^2 = 54 \Rightarrow \frac{x^2}{6} + \frac{(y+3)^2}{9} = 1$; this is an ellipse: the center is $(0, -3)$, the vertices are $(0, 0)$ and $(0, -6)$; $c = \sqrt{a^2 - b^2} = \sqrt{9 - 6} = \sqrt{3} \Rightarrow$ the foci are $\left(0, -3 \pm \sqrt{3}\right)$

63. $x^2 + 2y^2 - 2x - 4y = -1 \Rightarrow x^2 - 2x + 1 + 2\left(y^2 - 2y + 1\right) = 2 \Rightarrow (x - 1)^2 + 2(y - 1)^2 = 2$ $\Rightarrow \frac{(x-1)^2}{2} + (y - 1)^2 = 1$; this is an ellipse: the center is $(1, 1)$, the vertices are $\left(1 \pm \sqrt{2}, 1\right)$; $c = \sqrt{a^2 - b^2} = \sqrt{2 - 1} = 1 \Rightarrow$ the foci are $(2, 1)$ and $(0, 1)$

64. $4x^2 + y^2 + 8x - 2y = -1 \Rightarrow 4\left(x^2 + 2x + 1\right) + y^2 - 2y + 1 = 4 \Rightarrow 4(x + 1)^2 + (y - 1)^2 = 4$ $\Rightarrow (x + 1)^2 + \frac{(y-1)^2}{4} = 1$; this is an ellipse: the center is $(-1, 1)$, the vertices are $(-1, 3)$ and $(-1, -1)$; $c = \sqrt{a^2 - b^2} = \sqrt{4 - 1} = \sqrt{3} \Rightarrow$ the foci are $\left(-1, 1 \pm \sqrt{3}\right)$

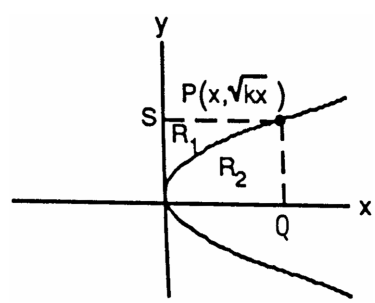
65. $x^2 - y^2 - 2x + 4y = 4 \Rightarrow x^2 - 2x + 1 - (y^2 - 4y + 4) = 1 \Rightarrow (x-1)^2 - (y-2)^2 = 1$; this is a hyperbola: the center is $(1, 2)$, the vertices are $(2, 2)$ and $(0, 2)$; $c = \sqrt{a^2 + b^2} = \sqrt{1+1} = \sqrt{2} \Rightarrow$ the foci are $\left(1 \pm \sqrt{2}, 2\right)$; the asymptotes are $y - 2 = \pm (x - 1)$

66. $x^2 - y^2 + 4x - 6y = 6 \Rightarrow x^2 + 4x + 4 - (y^2 + 6y + 9) = 1 \Rightarrow (x+2)^2 - (y+3)^2 = 1$; this is a hyperbola: the center is $(-2, -3)$, the vertices are $(-1, -3)$ and $(-3, -3)$; $c = \sqrt{a^2 + b^2} = \sqrt{1+1} = \sqrt{2} \Rightarrow$ the foci are $\left(-2 \pm \sqrt{2}, -3\right)$; the asymptotes are $y + 3 = \pm (x + 2)$

67. $2x^2 - y^2 + 6y = 3 \Rightarrow 2x^2 - (y^2 - 6y + 9) = -6 \Rightarrow \frac{(y-3)^2}{6} - \frac{x^2}{3} = 1$; this is a hyperbola: the center is $(0, 3)$, the vertices are $\left(0, 3 \pm \sqrt{6}\right)$; $c = \sqrt{a^2 + b^2} = \sqrt{6+3} = 3 \Rightarrow$ the foci are $(0, 6)$ and $(0, 0)$; the asymptotes are $\frac{y-3}{\sqrt{6}} = \pm \frac{x}{\sqrt{3}} \Rightarrow y = \pm \sqrt{2}x + 3$

68. $y^2 - 4x^2 + 16x = 24 \Rightarrow y^2 - 4(x^2 - 4x + 4) = 8 \Rightarrow \frac{y^2}{8} - \frac{(x-2)^2}{2} = 1$; this is a hyperbola: the center is $(2, 0)$, the vertices are $\left(2, \pm \sqrt{8}\right)$; $c = \sqrt{a^2 + b^2} = \sqrt{8+2} = \sqrt{10} \Rightarrow$ the foci are $\left(2, \pm \sqrt{10}\right)$; the asymptotes are $\frac{y}{\sqrt{8}} = \pm \frac{x-2}{\sqrt{2}} \Rightarrow y = \pm 2(x - 2)$

69. (a) $y^2 = kx \Rightarrow x = \frac{y^2}{k}$; the volume of the solid formed by revolving R_1 about the y-axis is $V_1 = \int_0^{\sqrt{kx}} \pi \left(\frac{y^2}{k}\right)^2 dy = \frac{\pi}{k^2} \int_0^{\sqrt{kx}} y^4\, dy = \frac{\pi x^2 \sqrt{kx}}{5}$; the volume of the right circular cylinder formed by revolving PQ about the y-axis is $V_2 = \pi x^2 \sqrt{kx} \Rightarrow$ the volume of the solid formed by revolving R_2 about the y-axis is $V_3 = V_2 - V_1 = \frac{4\pi x^2 \sqrt{kx}}{5}$. Therefore we can see the ratio of V_3 to V_1 is 4:1.



(b) The volume of the solid formed by revolving R_2 about the x-axis is $V_1 = \int_0^x \pi \left(\sqrt{kt}\right)^2 dt = \pi k \int_0^x t\, dt = \frac{\pi k x^2}{2}$. The volume of the right circular cylinder formed by revolving PS about the x-axis is $V_2 = \pi \left(\sqrt{kx}\right)^2 x = \pi k x^2 \Rightarrow$ the volume of the solid formed by revolving R_1 about the x-axis is $V_3 = V_2 - V_1 = \pi k x^2 - \frac{\pi k x^2}{2} = \frac{\pi k x^2}{2}$. Therefore the ratio of V_3 to V_1 is 1:1.

70. $y = \int \frac{w}{H} x\, dx = \frac{w}{H}\left(\frac{x^2}{2}\right) + C = \frac{wx^2}{2H} + C$; $y = 0$ when $x = 0 \Rightarrow 0 = \frac{w(0)^2}{2H} + C \Rightarrow C = 0$; therefore $y = \frac{wx^2}{2H}$ is the equation of the cable's curve

71. $x^2 = 4py$ and $y = p \Rightarrow x^2 = 4p^2 \Rightarrow x = \pm 2p$. Therefore the line $y = p$ cuts the parabola at points $(-2p, p)$ and $(2p, p)$, and these points are $\sqrt{[2p - (-2p)]^2 + (p - p)^2} = 4p$ units apart.

72. $\lim\limits_{x \to \infty} \left(\frac{b}{a} x - \frac{b}{a}\sqrt{x^2 - a^2}\right) = \frac{b}{a} \lim\limits_{x \to \infty} \left(x - \sqrt{x^2 - a^2}\right) = \frac{b}{a} \lim\limits_{x \to \infty} \left[\frac{\left(x - \sqrt{x^2 - a^2}\right)\left(x + \sqrt{x^2 - a^2}\right)}{x + \sqrt{x^2 - a^2}}\right]$

$= \frac{b}{a} \lim\limits_{x \to \infty} \left[\frac{x^2 - (x^2 - a^2)}{x + \sqrt{x^2 - a^2}}\right] = \frac{b}{a} \lim\limits_{x \to \infty} \left[\frac{a^2}{x + \sqrt{x^2 - a^2}}\right] = 0$

73. Let $y = \sqrt{1 - \frac{x^2}{4}}$ on the interval $0 \le x \le 2$. The area of the inscribed rectangle is given by
$A(x) = 2x\left(2\sqrt{1 - \frac{x^2}{4}}\right) = 4x\sqrt{1 - \frac{x^2}{4}}$ (since the length is 2x and the height is 2y)
$\Rightarrow A'(x) = 4\sqrt{1 - \frac{x^2}{4}} - \frac{x^2}{\sqrt{1 - \frac{x^2}{4}}}$. Thus $A'(x) = 0 \Rightarrow 4\sqrt{1 - \frac{x^2}{4}} - \frac{x^2}{\sqrt{1 - \frac{x^2}{4}}} = 0 \Rightarrow 4\left(1 - \frac{x^2}{4}\right) - x^2 = 0 \Rightarrow x^2 = 2$
$\Rightarrow x = \sqrt{2}$ (only the positive square root lies in the interval). Since $A(0) = A(2) = 0$ we have that $A\left(\sqrt{2}\right) = 4$ is the maximum area when the length is $2\sqrt{2}$ and the height is $\sqrt{2}$.

74. (a) Around the x-axis: $9x^2 + 4y^2 = 36 \Rightarrow y^2 = 9 - \frac{9}{4}x^2 \Rightarrow y = \pm\sqrt{9 - \frac{9}{4}x^2}$ and we use the positive root
$\Rightarrow V = 2\int_0^2 \pi\left(\sqrt{9 - \frac{9}{4}x^2}\right)^2 dx = 2\int_0^2 \pi\left(9 - \frac{9}{4}x^2\right) dx = 2\pi\left[9x - \frac{3}{4}x^3\right]_0^2 = 24\pi$
(b) Around the y-axis: $9x^2 + 4y^2 = 36 \Rightarrow x^2 = 4 - \frac{4}{9}y^2 \Rightarrow x = \pm\sqrt{4 - \frac{4}{9}y^2}$ and we use the positive root
$\Rightarrow V = 2\int_0^3 \pi\left(\sqrt{4 - \frac{4}{9}y^2}\right)^2 dy = 2\int_0^3 \pi\left(4 - \frac{4}{9}y^2\right) dy = 2\pi\left[4y - \frac{4}{27}y^3\right]_0^3 = 16\pi$

75. $9x^2 - 4y^2 = 36 \Rightarrow y^2 = \frac{9x^2 - 36}{4} \Rightarrow y = \pm\frac{3}{2}\sqrt{x^2 - 4}$ on the interval $2 \le x \le 4 \Rightarrow V = \int_2^4 \pi\left(\frac{3}{2}\sqrt{x^2 - 4}\right)^2 dx$
$= \frac{9\pi}{4}\int_2^4 (x^2 - 4)\, dx = \frac{9\pi}{4}\left[\frac{x^3}{3} - 4x\right]_2^4 = \frac{9\pi}{4}\left[\left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right)\right] = \frac{9\pi}{4}\left(\frac{56}{3} - 8\right) = \frac{3\pi}{4}(56 - 24) = 24\pi$

76. Let $P_1(-p, y_1)$ be any point on $x = -p$, and let $P(x, y)$ be a point where a tangent intersects $y^2 = 4px$. Now
$y^2 = 4px \Rightarrow 2y\frac{dy}{dx} = 4p \Rightarrow \frac{dy}{dx} = \frac{2p}{y}$; then the slope of a tangent line from P_1 is $\frac{y - y_1}{x - (-p)} = \frac{dy}{dx} = \frac{2p}{y}$
$\Rightarrow y^2 - yy_1 = 2px + 2p^2$. Since $x = \frac{y^2}{4p}$, we have $y^2 - yy_1 = 2p\left(\frac{y^2}{4p}\right) + 2p^2 \Rightarrow y^2 - yy_1 = \frac{1}{2}y^2 + 2p^2$
$\Rightarrow \frac{1}{2}y^2 - yy_1 - 2p^2 = 0 \Rightarrow y = \frac{2y_1 \pm \sqrt{4y_1^2 + 16p^2}}{2} = y_1 \pm \sqrt{y_1^2 + 4p^2}$. Therefore the slopes of the two tangents from P_1 are $m_1 = \frac{2p}{y_1 + \sqrt{y_1^2 + 4p^2}}$ and $m_2 = \frac{2p}{y_1 - \sqrt{y_1^2 + 4p^2}} \Rightarrow m_1 m_2 = \frac{4p^2}{y_1^2 - (y_1^2 + 4p^2)} = -1$
$\Rightarrow$ the lines are perpendicular

77. $(x - 2)^2 + (y - 1)^2 = 5 \Rightarrow 2(x - 2) + 2(y - 1)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-2}{y-1}$; $y = 0 \Rightarrow (x - 2)^2 + (0 - 1)^2 = 5$
$\Rightarrow (x - 2)^2 = 4 \Rightarrow x = 4$ or $x = 0 \Rightarrow$ the circle crosses the x-axis at $(4, 0)$ and $(0, 0)$; $x = 0$
$\Rightarrow (0 - 2)^2 + (y - 1)^2 = 5 \Rightarrow (y - 1)^2 = 1 \Rightarrow y = 2$ or $y = 0 \Rightarrow$ the circle crosses the y-axis at $(0, 2)$ and $(0, 0)$.
At $(4, 0)$: $\frac{dy}{dx} = -\frac{4-2}{0-1} = 2 \Rightarrow$ the tangent line is $y = 2(x - 4)$ or $y = 2x - 8$
At $(0, 0)$: $\frac{dy}{dx} = -\frac{0-2}{0-1} = -2 \Rightarrow$ the tangent line is $y = -2x$
At $(0, 2)$: $\frac{dy}{dx} = -\frac{0-2}{2-1} = 2 \Rightarrow$ the tangent line is $y - 2 = 2x$ or $y = 2x + 2$

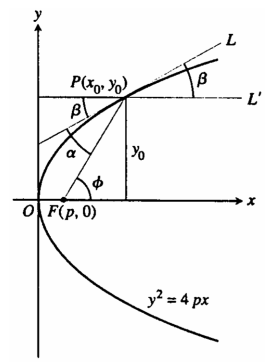
78. $x^2 - y^2 = 1 \Rightarrow x = \pm\sqrt{1 + y^2}$ on the interval $-3 \le y \le 3 \Rightarrow V = \int_{-3}^3 \pi\left(\sqrt{1 + y^2}\right)^2 dy = 2\int_0^3 \pi\left(\sqrt{1 + y^2}\right)^2 dy$
$= 2\pi\int_0^3 (1 + y^2)\, dy = 2\pi\left[y + \frac{y^3}{3}\right]_0^3 = 24\pi$

79. Let $y = \sqrt{16 - \frac{16}{9}x^2}$ on the interval $-3 \le x \le 3$. Since the plate is symmetric about the y-axis, $\overline{x} = 0$. For a vertical strip: $(\tilde{x}, \tilde{y}) = \left(x, \frac{\sqrt{16 - \frac{16}{9}x^2}}{2}\right)$, length $= \sqrt{16 - \frac{16}{9}x^2}$, width $= dx \Rightarrow$ area $= dA = \sqrt{16 - \frac{16}{9}x^2}\, dx$
$\Rightarrow$ mass $= dm = \delta\, dA = \delta\sqrt{16 - \frac{16}{9}x^2}\, dx$. Moment of the strip about the x-axis:
$\tilde{y}\, dm = \frac{\sqrt{16 - \frac{16}{9}x^2}}{2}\left(\delta\sqrt{16 - \frac{16}{9}x^2}\right) dx = \delta\left(8 - \frac{8}{9}x^2\right) dx$ so the moment of the plate about the x-axis is

$M_x = \int \tilde{y}\, dm = \int_{-3}^{3} \delta\left(8 - \frac{8}{9}x^2\right) dx = \delta\left[8x - \frac{8}{27}x^3\right]_{-3}^{3} = 32\delta$; also the mass of the plate is
$M = \int_{-3}^{3} \delta\sqrt{16 - \frac{16}{9}x^2}\, dx = \int_{-3}^{3} 4\delta\sqrt{1 - \left(\frac{1}{3}x\right)^2}\, dx = 4\delta\int_{-1}^{1} 3\sqrt{1 - u^2}\, du$ where $u = \frac{x}{3} \Rightarrow 3\, du = dx$; $x = -3$
$\Rightarrow u = -1$ and $x = 3 \Rightarrow u = 1$. Hence, $4\delta\int_{-1}^{1} 3\sqrt{1 - u^2}\, du = 12\delta\int_{-1}^{1}\sqrt{1 - u^2}\, du$
$= 12\delta\left[\frac{1}{2}\left(u\sqrt{1 - u^2} + \sin^{-1} u\right)\right]_{-1}^{1} = 6\pi\delta \Rightarrow \bar{y} = \frac{M_x}{M} = \frac{32\delta}{6\pi\delta} = \frac{16}{3\pi}$. Therefore the center of mass is $\left(0, \frac{16}{3\pi}\right)$.

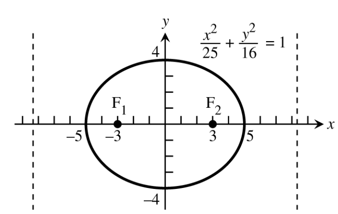
80. $y = \sqrt{x^2 + 1} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(x^2 + 1)^{-1/2}(2x) = \frac{x}{\sqrt{x^2 + 1}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{x^2 + 1} \Rightarrow \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{x^2}{x^2 + 1}}$
$= \sqrt{\frac{2x^2 + 1}{x^2 + 1}} \Rightarrow S = \int_0^{\sqrt{2}} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^{\sqrt{2}} 2\pi\sqrt{x^2 + 1}\sqrt{\frac{2x^2 + 1}{x^2 + 1}}\, dx = \int_0^{\sqrt{2}} 2\pi\sqrt{2x^2 + 1}\, dx$;
$\begin{bmatrix} u = \sqrt{2}x \\ du = \sqrt{2}\, dx \end{bmatrix} \to \frac{2\pi}{\sqrt{2}}\int_0^2 \sqrt{u^2 + 1}\, du = \frac{2\pi}{\sqrt{2}}\left[\frac{1}{2}\left(u\sqrt{u^2 + 1} + \ln\left(u + \sqrt{u^2 + 1}\right)\right)\right]_0^2 = \frac{\pi}{\sqrt{2}}\left[2\sqrt{5} + \ln\left(2 + \sqrt{5}\right)\right]$

81. (a) $\tan\beta = m_L \Rightarrow \tan\beta = f'(x_0)$ where $f(x) = \sqrt{4px}$;
$f'(x) = \frac{1}{2}(4px)^{-1/2}(4p) = \frac{2p}{\sqrt{4px}} \Rightarrow f'(x_0) = \frac{2p}{\sqrt{4px_0}}$
$= \frac{2p}{y_0} \Rightarrow \tan\beta = \frac{2p}{y_0}$.
(b) $\tan\phi = m_{FP} = \frac{y_0 - 0}{x_0 - p} = \frac{y_0}{x_0 - p}$
(c) $\tan\alpha = \frac{\tan\phi - \tan\beta}{1 + \tan\phi\tan\beta} = \frac{\left(\frac{y_0}{x_0 - p} - \frac{2p}{y_0}\right)}{1 + \left(\frac{y_0}{x_0 - p}\right)\left(\frac{2p}{y_0}\right)}$
$= \frac{y_0^2 - 2p(x_0 - p)}{y_0(x_0 - p + 2p)} = \frac{4px_0 - 2px_0 + 2p^2}{y_0(x_0 + p)} = \frac{2p(x_0 + p)}{y_0(x_0 + p)} = \frac{2p}{y_0}$

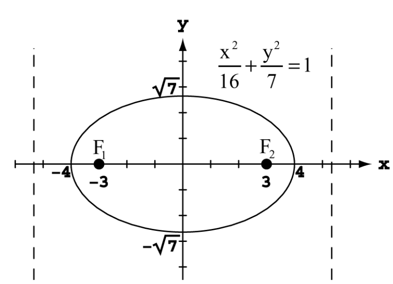


11.7 CONICS IN POLAR COORDINATES

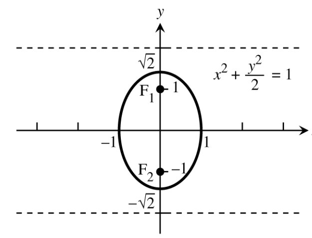
1. $16x^2 + 25y^2 = 400 \Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{25 - 16} = 3 \Rightarrow e = \frac{c}{a} = \frac{3}{5}$; $F(\pm 3, 0)$;
directrices are $x = 0 \pm \frac{a}{e} = \pm\frac{5}{\left(\frac{3}{5}\right)} = \pm\frac{25}{3}$



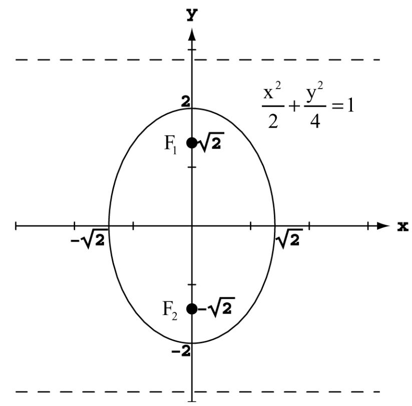
2. $7x^2 + 16y^2 = 112 \Rightarrow \frac{x^2}{16} + \frac{y^2}{7} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{16 - 7} = 3 \Rightarrow e = \frac{c}{a} = \frac{3}{4}$; $F(\pm 3, 0)$;
directrices are $x = 0 \pm \frac{a}{e} = \pm\frac{4}{\left(\frac{3}{4}\right)} = \pm\frac{16}{3}$



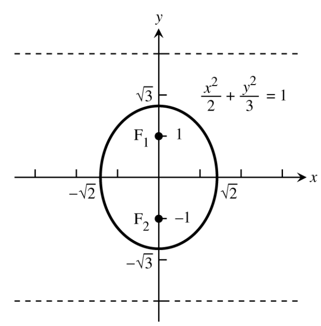
3. $2x^2 + y^2 = 2 \Rightarrow x^2 + \frac{y^2}{2} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{2 - 1} = 1 \Rightarrow e = \frac{c}{a} = \frac{1}{\sqrt{2}}$; $F(0, \pm 1)$;
directrices are $y = 0 \pm \frac{a}{e} = \pm \frac{\sqrt{2}}{\left(\frac{1}{\sqrt{2}}\right)} = \pm 2$



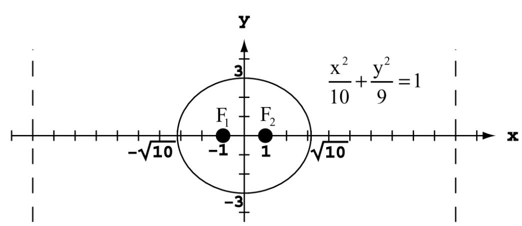
4. $2x^2 + y^2 = 4 \Rightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{4 - 2} = \sqrt{2} \Rightarrow e = \frac{c}{a} = \frac{\sqrt{2}}{2}$; $F\left(0, \pm \sqrt{2}\right)$;
directrices are $y = 0 \pm \frac{a}{e} = \pm \frac{2}{\left(\frac{\sqrt{2}}{2}\right)} = \pm 2\sqrt{2}$



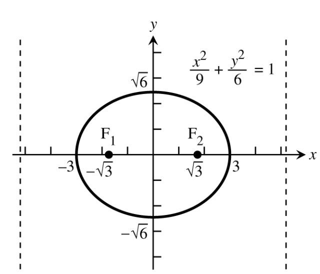
5. $3x^2 + 2y^2 = 6 \Rightarrow \frac{x^2}{2} + \frac{y^2}{3} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{3 - 2} = 1 \Rightarrow e = \frac{c}{a} = \frac{1}{\sqrt{3}}$; $F(0, \pm 1)$;
directrices are $y = 0 \pm \frac{a}{e} = \pm \frac{\sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)} = \pm 3$



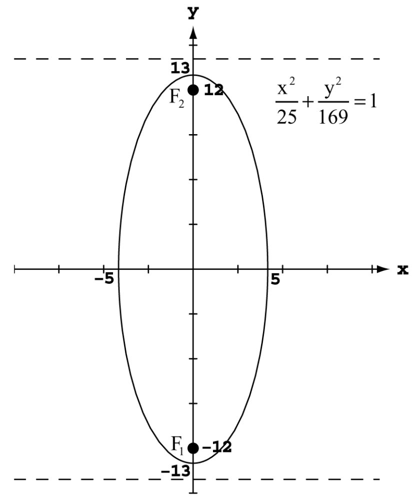
6. $9x^2 + 10y^2 = 90 \Rightarrow \frac{x^2}{10} + \frac{y^2}{9} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{10 - 9} = 1 \Rightarrow e = \frac{c}{a} = \frac{1}{\sqrt{10}}$; $F(\pm 1, 0)$;
directrices are $x = 0 \pm \frac{a}{e} = \pm \frac{\sqrt{10}}{\left(\frac{1}{\sqrt{10}}\right)} = \pm 10$



7. $6x^2 + 9y^2 = 54 \Rightarrow \frac{x^2}{9} + \frac{y^2}{6} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{9 - 6} = \sqrt{3} \Rightarrow e = \frac{c}{a} = \frac{\sqrt{3}}{3}$; $F\left(\pm \sqrt{3}, 0\right)$;
directrices are $x = 0 \pm \frac{a}{e} = \pm \frac{3}{\left(\frac{\sqrt{3}}{3}\right)} = \pm 3\sqrt{3}$



8. $169x^2 + 25y^2 = 4225 \Rightarrow \frac{x^2}{25} + \frac{y^2}{169} = 1 \Rightarrow c = \sqrt{a^2 - b^2}$
$= \sqrt{169 - 25} = 12 \Rightarrow e = \frac{c}{a} = \frac{12}{13}$; $F(0, \pm 12)$;
directrices are $y = 0 \pm \frac{a}{e} = \pm \frac{13}{\left(\frac{12}{13}\right)} = \pm \frac{169}{12}$



9. Foci: $(0, \pm 3)$, $e = 0.5 \Rightarrow c = 3$ and $a = \frac{c}{e} = \frac{3}{0.5} = 6 \Rightarrow b^2 = 36 - 9 = 27 \Rightarrow \frac{x^2}{27} + \frac{y^2}{36} = 1$

10. Foci: $(\pm 8, 0)$, $e = 0.2 \Rightarrow c = 8$ and $a = \frac{c}{e} = \frac{8}{0.2} = 40 \Rightarrow b^2 = 1600 - 64 = 1536 \Rightarrow \frac{x^2}{1600} + \frac{y^2}{1536} = 1$

11. Vertices: $(0, \pm 70)$, $e = 0.1 \Rightarrow a = 70$ and $c = ae = 70(0.1) = 7 \Rightarrow b^2 = 4900 - 49 = 4851 \Rightarrow \frac{x^2}{4851} + \frac{y^2}{4900} = 1$

12. Vertices: $(\pm 10, 0)$, $e = 0.24 \Rightarrow a = 10$ and $c = ae = 10(0.24) = 2.4 \Rightarrow b^2 = 100 - 5.76 = 94.24 \Rightarrow \frac{x^2}{100} + \frac{y^2}{94.24} = 1$

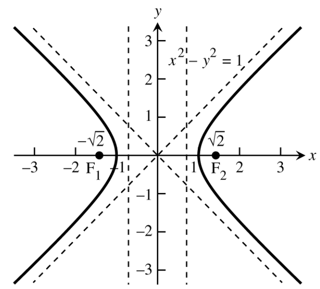
13. Focus: $\left(\sqrt{5}, 0\right)$, Directrix: $x = \frac{9}{\sqrt{5}} \Rightarrow c = ae = \sqrt{5}$ and $\frac{a}{e} = \frac{9}{\sqrt{5}} \Rightarrow \frac{ae}{e^2} = \frac{9}{\sqrt{5}} \Rightarrow \frac{\sqrt{5}}{e^2} = \frac{9}{\sqrt{5}} \Rightarrow e^2 = \frac{5}{9}$
$\Rightarrow e = \frac{\sqrt{5}}{3}$. Then $PF = \frac{\sqrt{5}}{3} PD \Rightarrow \sqrt{\left(x - \sqrt{5}\right)^2 + (y - 0)^2} = \frac{\sqrt{5}}{3}\left|x - \frac{9}{\sqrt{5}}\right| \Rightarrow \left(x - \sqrt{5}\right)^2 + y^2 = \frac{5}{9}\left(x - \frac{9}{\sqrt{5}}\right)^2$
$\Rightarrow x^2 - 2\sqrt{5}\,x + 5 + y^2 = \frac{5}{9}\left(x^2 - \frac{18}{\sqrt{5}}x + \frac{81}{5}\right) \Rightarrow \frac{4}{9}x^2 + y^2 = 4 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$

14. Focus: $(4, 0)$, Directrix: $x = \frac{16}{3} \Rightarrow c = ae = 4$ and $\frac{a}{e} = \frac{16}{3} \Rightarrow \frac{ae}{e^2} = \frac{16}{3} \Rightarrow \frac{4}{e^2} = \frac{16}{3} \Rightarrow e^2 = \frac{3}{4} \Rightarrow e = \frac{\sqrt{3}}{2}$. Then
$PF = \frac{\sqrt{3}}{2} PD \Rightarrow \sqrt{(x-4)^2 + (y-0)^2} = \frac{\sqrt{3}}{2}\left|x - \frac{16}{3}\right| \Rightarrow (x-4)^2 + y^2 = \frac{3}{4}\left(x - \frac{16}{3}\right)^2 \Rightarrow x^2 - 8x + 16 + y^2$
$= \frac{3}{4}\left(x^2 - \frac{32}{3}x + \frac{256}{9}\right) \Rightarrow \frac{1}{4}x^2 + y^2 = \frac{16}{3} \Rightarrow \frac{x^2}{\left(\frac{64}{3}\right)} + \frac{y^2}{\left(\frac{16}{3}\right)} = 1$

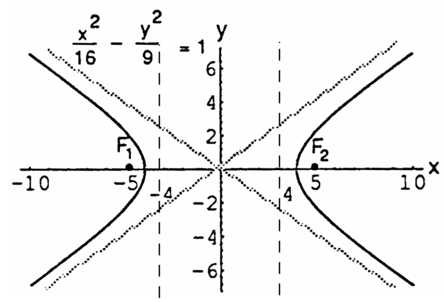
15. Focus: $(-4, 0)$, Directrix: $x = -16 \Rightarrow c = ae = 4$ and $\frac{a}{e} = 16 \Rightarrow \frac{ae}{e^2} = 16 \Rightarrow \frac{4}{e^2} = 16 \Rightarrow e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$. Then
$PF = \frac{1}{2} PD \Rightarrow \sqrt{(x+4)^2 + (y-0)^2} = \frac{1}{2}|x + 16| \Rightarrow (x+4)^2 + y^2 = \frac{1}{4}(x+16)^2 \Rightarrow x^2 + 8x + 16 + y^2$
$= \frac{1}{4}(x^2 + 32x + 256) \Rightarrow \frac{3}{4}x^2 + y^2 = 48 \Rightarrow \frac{x^2}{64} + \frac{y^2}{48} = 1$

16. Focus: $\left(-\sqrt{2}, 0\right)$, Directrix: $x = -2\sqrt{2} \Rightarrow c = ae = \sqrt{2}$ and $\frac{a}{e} = 2\sqrt{2} \Rightarrow \frac{ae}{e^2} = 2\sqrt{2} \Rightarrow \frac{\sqrt{2}}{e^2} = 2\sqrt{2} \Rightarrow e^2 = \frac{1}{2}$
$\Rightarrow e = \frac{1}{\sqrt{2}}$. Then $PF = \frac{1}{\sqrt{2}} PD \Rightarrow \sqrt{\left(x + \sqrt{2}\right)^2 + (y-0)^2} = \frac{1}{\sqrt{2}}\left|x + 2\sqrt{2}\right| \Rightarrow \left(x + \sqrt{2}\right)^2 + y^2$
$= \frac{1}{2}\left(x + 2\sqrt{2}\right)^2 \Rightarrow x^2 + 2\sqrt{2}\,x + 2 + y^2 = \frac{1}{2}\left(x^2 + 4\sqrt{2}\,x + 8\right) \Rightarrow \frac{1}{2}x^2 + y^2 = 2 \Rightarrow \frac{x^2}{4} + \frac{y^2}{2} = 1$

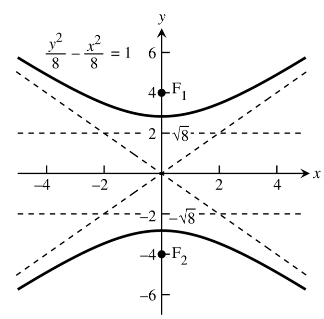
17. $x^2 - y^2 = 1 \Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{1+1} = \sqrt{2} \Rightarrow e = \frac{c}{a}$
$= \frac{\sqrt{2}}{1} = \sqrt{2}$; asymptotes are $y = \pm x$; $F\left(\pm\sqrt{2}, 0\right)$;
directrices are $x = 0 \pm \frac{a}{e} = \pm\frac{1}{\sqrt{2}}$



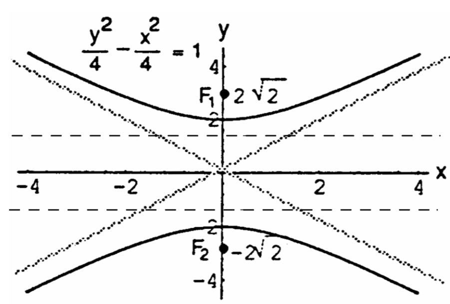
18. $9x^2 - 16y^2 = 144 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{16+9} = 5 \Rightarrow e = \frac{c}{a} = \frac{5}{4}$; asymptotes are
$y = \pm\frac{3}{4}x$; $F(\pm 5, 0)$; directrices are $x = 0 \pm \frac{a}{e}$
$= \pm\frac{16}{5}$



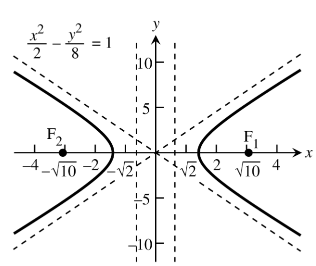
19. $y^2 - x^2 = 8 \Rightarrow \frac{y^2}{8} - \frac{x^2}{8} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{8+8} = 4 \Rightarrow e = \frac{c}{a} = \frac{4}{\sqrt{8}} = \sqrt{2}$; asymptotes are
$y = \pm x$; $F(0, \pm 4)$; directrices are $y = 0 \pm \frac{a}{e}$
$= \pm\frac{\sqrt{8}}{\sqrt{2}} = \pm 2$



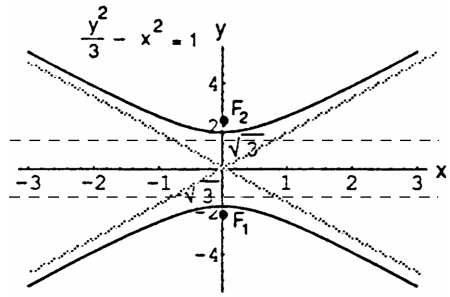
20. $y^2 - x^2 = 4 \Rightarrow \frac{y^2}{4} - \frac{x^2}{4} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{4+4} = 2\sqrt{2} \Rightarrow e = \frac{c}{a} = \frac{2\sqrt{2}}{2} = \sqrt{2}$; asymptotes
are $y = \pm x$; $F\left(0, \pm 2\sqrt{2}\right)$; directrices are $y = 0 \pm \frac{a}{e}$
$= \pm\frac{2}{\sqrt{2}} = \pm\sqrt{2}$



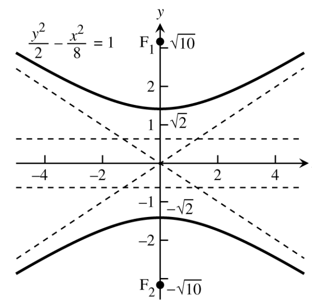
21. $8x^2 - 2y^2 = 16 \Rightarrow \frac{x^2}{2} - \frac{y^2}{8} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{2+8} = \sqrt{10} \Rightarrow e = \frac{c}{a} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$; asymptotes
are $y = \pm 2x$; $F\left(\pm\sqrt{10}, 0\right)$; directrices are $x = 0 \pm \frac{a}{e}$
$= \pm\frac{\sqrt{2}}{\sqrt{5}} = \pm\frac{2}{\sqrt{10}}$



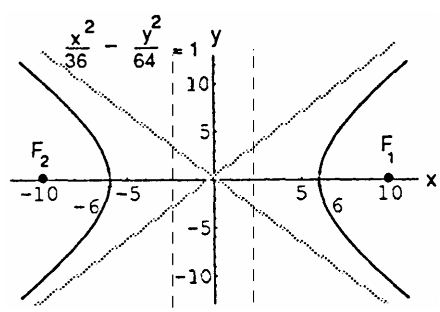
22. $y^2 - 3x^2 = 3 \Rightarrow \frac{y^2}{3} - x^2 = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{3+1} = 2 \Rightarrow e = \frac{c}{a} = \frac{2}{\sqrt{3}}$; asymptotes are
$y = \pm\sqrt{3}\,x$; $F(0, \pm 2)$; directrices are $y = 0 \pm \frac{a}{e}$
$= \pm\frac{\sqrt{3}}{\left(\frac{2}{\sqrt{3}}\right)} = \pm\frac{3}{2}$



23. $8y^2 - 2x^2 = 16 \Rightarrow \frac{y^2}{2} - \frac{x^2}{8} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{2+8} = \sqrt{10} \Rightarrow e = \frac{c}{a} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$; asymptotes
are $y = \pm\frac{x}{2}$; $F\left(0, \pm\sqrt{10}\right)$; directrices are $y = 0 \pm \frac{a}{e}$
$= \pm\frac{\sqrt{2}}{\sqrt{5}} = \pm\frac{2}{\sqrt{10}}$



24. $64x^2 - 36y^2 = 2304 \Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1 \Rightarrow c = \sqrt{a^2 + b^2}$
$= \sqrt{36+64} = 10 \Rightarrow e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$; asymptotes are
$y = \pm\frac{4}{3}x$; $F(\pm 10, 0)$; directrices are $x = 0 \pm \frac{a}{e}$
$= \pm\frac{6}{\left(\frac{5}{3}\right)} = \pm\frac{18}{5}$



25. Vertices $(0, \pm 1)$ and $e = 3 \Rightarrow a = 1$ and $e = \frac{c}{a} = 3 \Rightarrow c = 3a = 3 \Rightarrow b^2 = c^2 - a^2 = 9 - 1 = 8 \Rightarrow y^2 - \frac{x^2}{8} = 1$

26. Vertices $(\pm 2, 0)$ and $e = 2 \Rightarrow a = 2$ and $e = \frac{c}{a} = 2 \Rightarrow c = 2a = 4 \Rightarrow b^2 = c^2 - a^2 = 16 - 4 = 12 \Rightarrow \frac{x^2}{4} - \frac{y^2}{12} = 1$

27. Foci $(\pm 3, 0)$ and $e = 3 \Rightarrow c = 3$ and $e = \frac{c}{a} = 3 \Rightarrow c = 3a \Rightarrow a = 1 \Rightarrow b^2 = c^2 - a^2 = 9 - 1 = 8 \Rightarrow x^2 - \frac{y^2}{8} = 1$

28. Foci $(0, \pm 5)$ and $e = 1.25 \Rightarrow c = 5$ and $e = \frac{c}{a} = 1.25 = \frac{5}{4} \Rightarrow c = \frac{5}{4}a \Rightarrow 5 = \frac{5}{4}a \Rightarrow a = 4 \Rightarrow b^2 = c^2 - a^2$
$= 25 - 16 = 9 \Rightarrow \frac{y^2}{16} - \frac{x^2}{9} = 1$

29. $e = 1, x = 2 \Rightarrow k = 2 \Rightarrow r = \frac{2(1)}{1 + (1)\cos\theta} = \frac{2}{1+\cos\theta}$

30. $e = 1, y = 2 \Rightarrow k = 2 \Rightarrow r = \frac{2(1)}{1 + (1)\sin\theta} = \frac{2}{1+\sin\theta}$

31. $e = 5, y = -6 \Rightarrow k = 6 \Rightarrow r = \frac{6(5)}{1 - 5\sin\theta} = \frac{30}{1-5\sin\theta}$

32. $e = 2, x = 4 \Rightarrow k = 4 \Rightarrow r = \frac{4(2)}{1 + 2\cos\theta} = \frac{8}{1+2\cos\theta}$

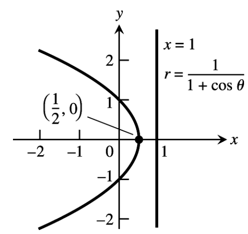
33. $e = \frac{1}{2}, x = 1 \Rightarrow k = 1 \Rightarrow r = \frac{\left(\frac{1}{2}\right)(1)}{1 + \left(\frac{1}{2}\right)\cos\theta} = \frac{1}{2+\cos\theta}$

34. $e = \frac{1}{4}, x = -2 \Rightarrow k = 2 \Rightarrow r = \frac{\left(\frac{1}{4}\right)(2)}{1 - \left(\frac{1}{4}\right)\cos\theta} = \frac{2}{4-\cos\theta}$

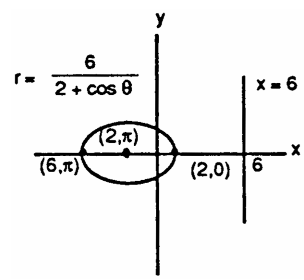
35. $e = \frac{1}{5}, y = -10 \Rightarrow k = 10 \Rightarrow r = \frac{\left(\frac{1}{5}\right)(10)}{1 - \left(\frac{1}{5}\right)\sin\theta} = \frac{10}{5-\sin\theta}$

36. $e = \frac{1}{3}, y = 6 \Rightarrow k = 6 \Rightarrow r = \frac{\left(\frac{1}{3}\right)(6)}{1 + \left(\frac{1}{3}\right)\sin\theta} = \frac{6}{3+\sin\theta}$

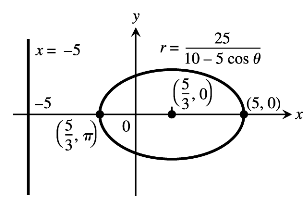
37. $r = \frac{1}{1+\cos\theta} \Rightarrow e = 1, k = 1 \Rightarrow x = 1$



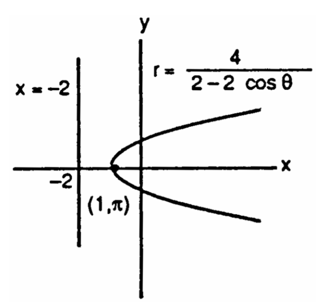
38. $r = \frac{6}{2+\cos\theta} = \frac{3}{1+\left(\frac{1}{2}\right)\cos\theta} \Rightarrow e = \frac{1}{2}, k = 6 \Rightarrow x = 6;$
$a(1 - e^2) = ke \Rightarrow a\left[1 - \left(\frac{1}{2}\right)^2\right] = 3 \Rightarrow \frac{3}{4}a = 3$
$\Rightarrow a = 4 \Rightarrow ea = 2$



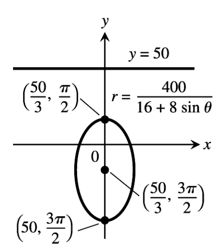
39. $r = \frac{25}{10-5\cos\theta} \Rightarrow r = \frac{\left(\frac{25}{10}\right)}{1 - \left(\frac{5}{10}\right)\cos\theta} = \frac{\left(\frac{5}{2}\right)}{1 - \left(\frac{1}{2}\right)\cos\theta}$
$\Rightarrow e = \frac{1}{2}, k = 5 \Rightarrow x = -5; a(1 - e^2) = ke$
$\Rightarrow a\left[1 - \left(\frac{1}{2}\right)^2\right] = \frac{5}{2} \Rightarrow \frac{3}{4}a = \frac{5}{2} \Rightarrow a = \frac{10}{3} \Rightarrow ea = \frac{5}{3}$



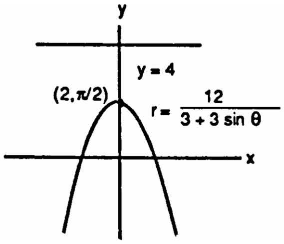
40. $r = \frac{4}{2-2\cos\theta} \Rightarrow r = \frac{2}{1-\cos\theta} \Rightarrow e = 1, k = 2 \Rightarrow x = -2$



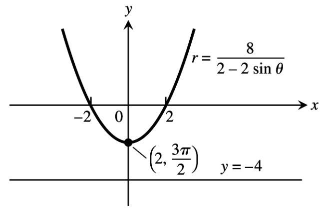
41. $r = \frac{400}{16+8\sin\theta} \Rightarrow r = \frac{\left(\frac{400}{16}\right)}{1 + \left(\frac{8}{16}\right)\sin\theta} \Rightarrow r = \frac{25}{1+\left(\frac{1}{2}\right)\sin\theta}$
$e = \frac{1}{2}, k = 50 \Rightarrow y = 50; a(1 - e^2) = ke$
$\Rightarrow a\left[1 - \left(\frac{1}{2}\right)^2\right] = 25 \Rightarrow \frac{3}{4}a = 25 \Rightarrow a = \frac{100}{3}$
$\Rightarrow ea = \frac{50}{3}$



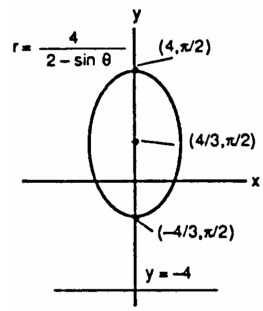
42. $r = \frac{12}{3+3\sin\theta} \Rightarrow r = \frac{4}{1+\sin\theta} \Rightarrow e = 1,$
$k = 4 \Rightarrow y = 4$



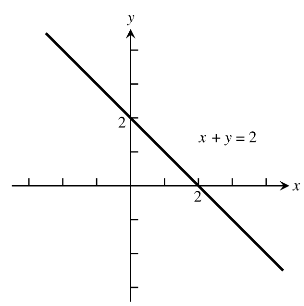
43. $r = \frac{8}{2-2\sin\theta} \Rightarrow r = \frac{4}{1-\sin\theta} \Rightarrow e = 1,$
$k = 4 \Rightarrow y = -4$



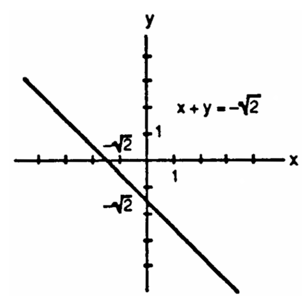
44. $r = \frac{4}{2-\sin\theta} \Rightarrow r = \frac{2}{1-\left(\frac{1}{2}\right)\sin\theta} \Rightarrow e = \frac{1}{2}, k = 4$
$\Rightarrow y = -4;\ a\left(1-e^2\right) = ke \Rightarrow a\left[1-\left(\frac{1}{2}\right)^2\right] = 2$
$\Rightarrow \frac{3}{4}a = 2 \Rightarrow a = \frac{8}{3} \Rightarrow ea = \frac{4}{3}$



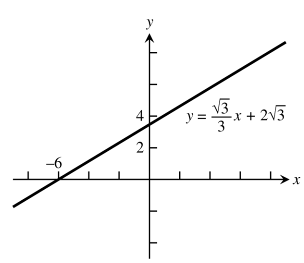
45. $r\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2} \Rightarrow r\left(\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right)$
$= \sqrt{2} \Rightarrow \frac{1}{\sqrt{2}}r\cos\theta + \frac{1}{\sqrt{2}}r\sin\theta = \sqrt{2} \Rightarrow \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$
$= \sqrt{2} \Rightarrow x + y = 2 \Rightarrow y = 2 - x$



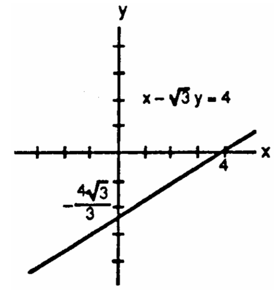
46. $r\cos\left(\theta + \frac{3\pi}{4}\right) = 1 \Rightarrow r\left(\cos\theta\cos\frac{3\pi}{4} - \sin\theta\sin\frac{3\pi}{4}\right) = 1$
$\Rightarrow -\frac{\sqrt{2}}{2}r\cos\theta - \frac{\sqrt{2}}{2}r\sin\theta = 1 \Rightarrow x + y = -\sqrt{2}$
$\Rightarrow y = -x - \sqrt{2}$



47. $r\cos\left(\theta - \frac{2\pi}{3}\right) = 3 \Rightarrow r\left(\cos\theta\cos\frac{2\pi}{3} + \sin\theta\sin\frac{2\pi}{3}\right) = 3$
$\Rightarrow -\frac{1}{2}r\cos\theta + \frac{\sqrt{3}}{2}r\sin\theta = 3 \Rightarrow -\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 3$
$\Rightarrow -x + \sqrt{3}y = 6 \Rightarrow y = \frac{\sqrt{3}}{3}x + 2\sqrt{3}$



48. $r\cos\left(\theta+\frac{\pi}{3}\right)=2 \Rightarrow r\left(\cos\theta\cos\frac{\pi}{3}-\sin\theta\sin\frac{\pi}{3}\right)=2$
$\Rightarrow \frac{1}{2}r\cos\theta-\frac{\sqrt{3}}{2}r\sin\theta=2 \Rightarrow \frac{1}{2}x-\frac{\sqrt{3}}{2}y=2$
$\Rightarrow x-\sqrt{3}y=4 \Rightarrow y=\frac{\sqrt{3}}{3}x-\frac{4\sqrt{3}}{3}$



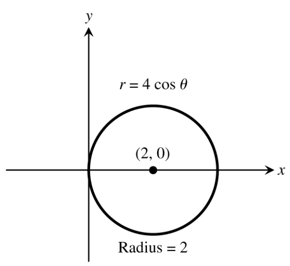
49. $\sqrt{2}x+\sqrt{2}y=6 \Rightarrow \sqrt{2}r\cos\theta+\sqrt{2}r\sin\theta=6 \Rightarrow r\left(\frac{\sqrt{2}}{2}\cos\theta+\frac{\sqrt{2}}{2}\sin\theta\right)=3 \Rightarrow r\left(\cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta\right)$
$=3 \Rightarrow r\cos\left(\theta-\frac{\pi}{4}\right)=3$

50. $\sqrt{3}x-y=1 \Rightarrow \sqrt{3}r\cos\theta-r\sin\theta=1 \Rightarrow r\left(\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)=\frac{1}{2} \Rightarrow r\left(\cos\frac{\pi}{6}\cos\theta-\sin\frac{\pi}{6}\sin\theta\right)$
$=\frac{1}{2} \Rightarrow r\cos\left(\theta+\frac{\pi}{6}\right)=\frac{1}{2}$

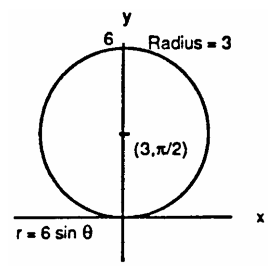
51. $y=-5 \Rightarrow r\sin\theta=-5 \Rightarrow -r\sin\theta=5 \Rightarrow r\sin(-\theta)=5 \Rightarrow r\cos\left(\frac{\pi}{2}-(-\theta)\right)=5 \Rightarrow r\cos\left(\theta+\frac{\pi}{2}\right)=5$

52. $x=-4 \Rightarrow r\cos\theta=-4 \Rightarrow -r\cos\theta=4 \Rightarrow r\cos(\theta-\pi)=4$

53.



54.



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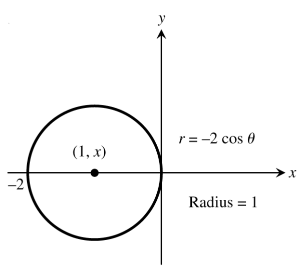

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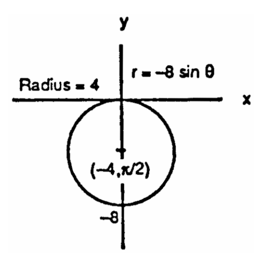
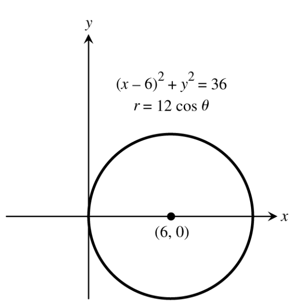
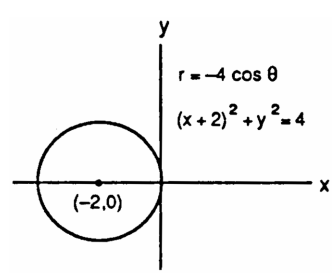


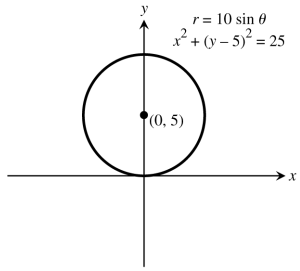
57. $(x-6)^2+y^2=36 \Rightarrow C=(6,0),\ a=6$
$\Rightarrow r=12\cos\theta$ is the polar equation



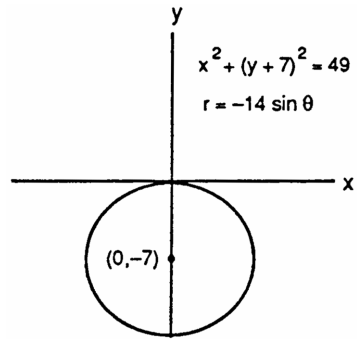
58. $(x+2)^2+y^2=4 \Rightarrow C=(-2,0),\ a=2$
$\Rightarrow r=-4\cos\theta$ is the polar equation



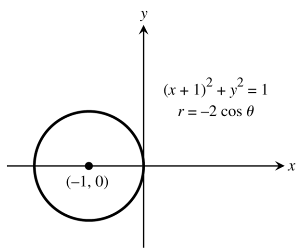
59. $x^2 + (y - 5)^2 = 25 \Rightarrow C = (0, 5), a = 5$
$\Rightarrow r = 10 \sin\theta$ is the polar equation



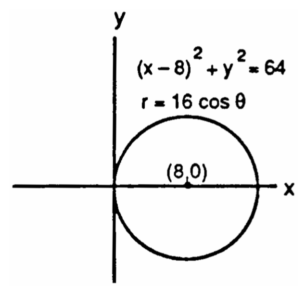
60. $x^2 + (y + 7)^2 = 49 \Rightarrow C = (0, -7), a = 7$
$\Rightarrow r = -14 \sin\theta$ is the polar equation



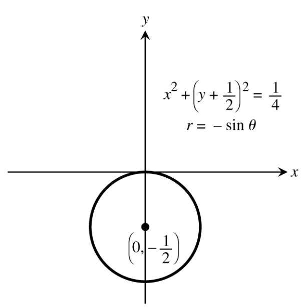
61. $x^2 + 2x + y^2 = 0 \Rightarrow (x + 1)^2 + y^2 = 1$
$\Rightarrow C = (-1, 0), a = 1 \Rightarrow r = -2 \cos\theta$ is the polar equation



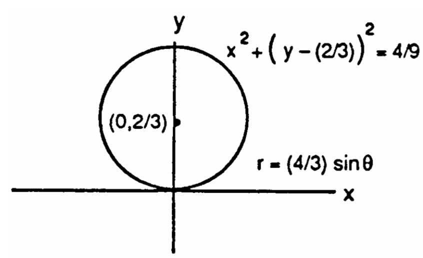
62. $x^2 - 16x + y^2 = 0 \Rightarrow (x - 8)^2 + y^2 = 64$
$\Rightarrow C = (8, 0), a = 8 \Rightarrow r = 16 \cos\theta$ is the polar equation



63. $x^2 + y^2 + y = 0 \Rightarrow x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$
$\Rightarrow C = \left(0, -\frac{1}{2}\right), a = \frac{1}{2} \Rightarrow r = -\sin\theta$ is the polar equation



64. $x^2 + y^2 - \frac{4}{3}y = 0 \Rightarrow x^2 + \left(y - \frac{2}{3}\right)^2 = \frac{4}{9}$
$\Rightarrow C = \left(0, \frac{2}{3}\right), a = \frac{2}{3} \Rightarrow r = \frac{4}{3}\sin\theta$ is the polar equation



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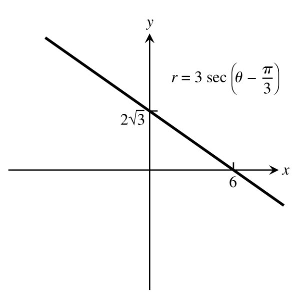

66.